ECE2049 LECTURE 1

TODAY

- INTRO TO NUMBER REPRESENTATIONS

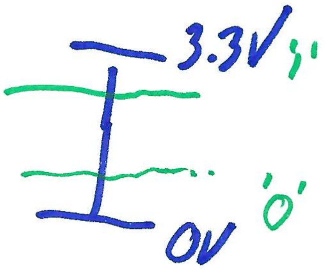
ADMINISTRIVIA

- HW1: ON WEBSITE AFTER CLASS
- MAKE SURE YOU CAN ACCESS
    - CANVAS
    - PIAZZA
    - DISCORD (INVITE ON CANVAS)
- IF YOU HAVE NOT DONE SO ALREADY...
    - ORDER YOUR LAB PARTS (SEE LABS PAGE ON WEBSITE)
        - WHEN YOU HAVE DONE SO, PLEASE FILL OUT THE "PARTS SURVEY"
    - FILL OUT THE "BACKGROUND [illegible] SURVEY" ON THE WEBSITE.

Module 1. Intro to Number Representations

Topics

- How do we store (or "encode") information in digital systems?
- Specifically: how do we store numbers?



First things first: Remembering Digital Logic

Before we can talk about how computing systems are built, we first need to talk about their basic building block: digital logic. In digital logic, information is represented in binary *bits*.

1 BIT = VALUE OF 0 OR 1

Digital logic defines how we can process information using bits:

- LOGIC (AND, OR, NOT) .
- ARITHMETIC (+, -, /...)

- STORAGE

⇒ COMBINE TO BUILD MORE COMPLEX COMPONENTS

First things first: *n* bits differentiate among 2^n things.

$N_{BITS} = 2^N$ UNIQUE "CODES"

EX. 2 BITS $2^2 = 4$ POSSIBLE CODES
00, 01, 10, 11

Terminology: 1 byte = 8 binary digits = 8 bits (e.g. 10010011)

½ byte = 1 nibble = 4 bits

1 word = 2 (or more) bytes --> MSP430 word = 2 bytes (16 BITS)

1 double word = 2 words (4 bytes on MSP430) (32 BITS)

In computers, ***information*** and ***memory space*** is organized in to multiples of bytes. But what do the bytes mean?

The meaning of bits and bytes assigned by convention!

>> Under a given coding convention, a byte can represent up to $2^8 = 256$ things

For example, 1 byte (8 bits) could encode:

- A letter in an alphabet

'D' ~~Ae~~ · ASCII

- One or more decimal numbers

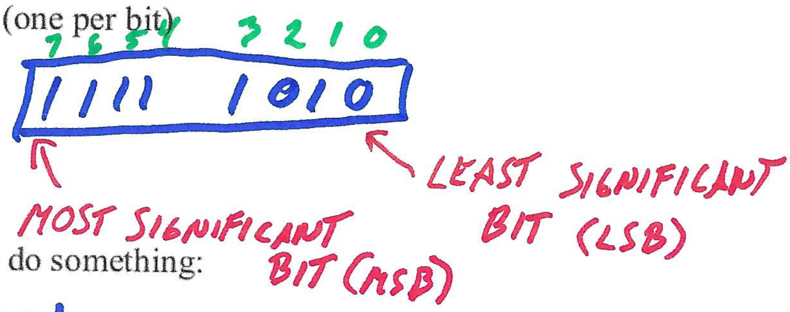
42, −42, −42.5

- The state of eight individual things (one per bit)

"~~B~~ BIT VECTOR"

BIT6 = 1

BIT2 = 0



- An *instruction* that tells the CPU to do something:

C3h => 1100 0011b

=> RET ← CPU INSTRUCTION TO RETURN FROM FUNCTION

We call these conventions **encoding formats**. They represent a kind of contract on how data will be stored and used. As programmers, it is up to us to assign meaning to those bits—which defines what operations we perform on them.

Conversion between Bases and Formats: Binary

Positional Number Systems

We write numbers in a positional system, which can be defined as:

BASE (RADIX)

DECIMAL $1734. \Rightarrow 1\times10^3 + 7\times10^2 + 3\times10^1 + 4\times10^0$

BASE 10

ddd.ddd

$D = \sum_{i=-N}^{P-1} d_i r^i$ WHERE

r = RADIX

d_i = iTH DIGIT

N = DIGITS LEFT OF

P = DIGITS RIGHT OF

For binary numbers, we can write this definition as:

BASE 2: 0-1

$B = \sum_{i=-N}^{P} b_i (2^i)$ WHERE $b_i \in \{0,1\}$

Unsigned integers = All bits used to convey magnitude (whole numbers ≥ 0)

Ex. $\overset{7}{1}001 . 000\overset{0}{1}$ ← LSB

MSB

$\Rightarrow 1\times2^7 + 0\times2^6 + 0\times2^5 + 1\times2^4 + 0\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$

$= 128 + 16 + 1 = \boxed{145d}$

Decimal to Binary Conversion – Successive Division

↳ DIVIDE BY 2, LOOK AT REMAINDER

Ex. 44

44/2 = 22 R 0 LSB

22/2 = 11 R 0

11/2 = 5 R 1

5/2 = 2 R 1

2/2 = 1 R 0

/2 = 0 R 1 MSB

$\boxed{101100b}$

Note: To differentiate numbers in different formats, we use notation to denote the radix used wo write it. For binary: 1010_2 or 1010b; decimal: 1010_{10} or 1010d (or just 1010)

Hexadecimal: A common way to write binary numbers

Since working in binary can be cumbersome, we often write numbers in *hexadecimal*, which is base 16.

Simple rule for conversion: ONE HEX "DIGIT" = 4 BITS

--> 1 Hex character represents values from 0 to 15d using digits 0 – Fh

DEC	BIN	HEX	DEC	BIN	HEX
0	0000	0	8	1000	8
1	0001	1	9	1001	9
2	0010	2	10	1010	A
3	0011	3	11	1011	B
4	0100	4	12	1100	C
5	0101	5	13	1101	D
6	0110	6	14	1110	E
7	0111	7	15	1111	F

Ex. 158d =

158/16 = 9 R14

9/16 = 0 R 9

= $\boxed{9Eh}$ = 0x9

If you memorize anything in this class, memorize these!

Notation: Numbers in hex are written as 1010h or 0x1010

Conversion between hex and binary is piece of cake! Just convert each hex digit to a binary nibble...

1001 1110b = 158d

9 Eh

1001 = $2^3 + 2^0 = 8+1 = 9$

2^3 2^2 2^1 2^0

8 4 2 1

Or vice versa:

8AC4h = 8 A C 4

$\boxed{1000 \quad 1010 \quad 1100 \quad 0100}$ b

Note: A modern computer always stores information in binary form. Writing in hex is just a faster way for us to read and write these numbers—the machine's representation is still binary.

How do we store negative numbers?

One way: Sign Magnitude integers = *n*-1 bits used to convey magnitude with "most significant bit" or MSB used for sign. Convention: 0 = +, 1 = -

M

S -

FOR 8 BITS: SMMM MMMM

0|010 0001 = +33
1|010 0001 = -33

TWO REPRESENTATIONS FOR ZERO!

0|000 0000 = 0
1|000 0000 = -0 ???

AMBIGUOUS

INEFFICIENT

Note: This format has 2 representations of 0 = +0 and -0 !

Another way: Two's Complement integers = More common format for signed integers. For *n* bits, values range from $-2^{(n-1)}$ to $2^{(n-1)}-1$

How it works:

Positive numbers: Follow same format as unsigned numbers

1026 = 0000 0100 0000 0010b = 0402h

$2^{10} + 2^{1} = 1024+2 = 1026$

ASSUME 8 BIT NUMBERS

Negative numbers: *Write* magnitude, *Complement* each bit, *Add* 1

-15 =

ONLY IF NEGATIVE

0000 1111 MAGNITUDE

1111 0000 COMPLEMENT (FLIP ALL BIT

\+ 1

1111 0001b

SIGN BIT IS SET

1+1 =10

1001
\+ 1
1010

Ex. -1

0000 0001
1111 1110
1
1111 1111 =? -1

IN REVERSE:

1111 0001
0000 1110
1
→0000 1111 = 15
-15

Range of Values for 2's Complement

For 16 BITS: (-2^{+15}) to $(2^{15}-1)$ = -32768 TO +32767

0111 1111 1111 1111b = +32767
0111 1111 1111 1110b = +32766
. . .

0000 0000 0000 0001b = 1
0000 0000 0000 0000b
1111 1111 1111 1111 = -1

. . .

1000 0000 0000 0001b
1000 0000 0000 0000b = -32768

Ex: Find the 8-bit two's complement representation of 104 and -80

Ex. 104 POSITIVE, SO TREAT IT LIKE UNSIGNED

104/2 = 52 R0

52/2 = 26 R0

⋮

$\boxed{0110\ 1000b}$

$= 2^6 + 2^5 + 2^3 = 104$

- 80 => NEGATIVE, NEED 2'S COMP PROCEDURE

80 => 0101 0000

1010 1111 COMP

\+ 1

$\boxed{1011\ 0000} = -80$

Ex: What are the decimal equivalent values of these 2's complement values

0010 0011b

POSITIVE

$2^5 + 2^1 + 2^0 = 32+2+1 = \boxed{35d}$

1000 0011b

NEGATIVE, NEED 2'S COMP PROCEDURE

1000 0011

0111 1100 COMP

\+ 1 ADD 1

$\boxed{0111\ 1101} = \boxed{-125}$

Ex: What decimal value does 8008h represent as an…
(a) unsigned integer (b) 2's comp integer (c) sign-magnitude integer

A. UNSIGNED

$$8008h = \overset{15}{1}000\ 0000\ 0000\ \overset{3}{1}00\overset{0}{0}\,b$$

$$= 2^{15} + 2^{3} = \boxed{32776}$$

B. NUMBER IS NEGATIVE, SO NEED 2'S COMP PROCEDURE

$$\begin{array}{rl} 1000\ 0000\ 0000\ 1000 & \\ 0111\ 1111\ 1111\ 0111 & \text{COMP} \\ +\ 0000\ 0000\ 0000\ 0001 & \\ \hline 0111\ 1111\ 1111\ 1000 & \end{array}$$

[illegible] $2^{14} + 2^{13} + \ldots + 2^{3} = \boxed{-32760}$

C. SIGN-MAGNITUDE

SIGN: −
MAG: 8 $\Rightarrow \boxed{-8}$

What about things that aren't integers?

Characters

To handle letters and other displayable characters, we need an encoding format to describe how we can represent these values in binary. One very common format for this is ASCII (American Standard Code for Information Exchange), which defines a table of binary codes that represent various characters.

ASCII = 7 OR 8 BIT ENCODING

IN PRACTICE: 1 CHARACTER = 1 BYTE.

'D' = 44h = 68d

Note: Other formats exists for representing different types of characters (alphabets and character sets for all human languages, emoji, etc.). For information on this, see "Unicode".

Unicode Examples

IN UNICODE, ONE CHARACTER = >1 BYTE

Unicode Name	Bit representation	Character
`U+00FC LATIN SMALL LETTER U WITH DIAERESIS`	`C3 BC`	ü
`U+1F602 FACE WITH TEARS OF JOY`	`F0 9f 98 82`	😂
`U+1F363 SUSHI`	`F0 9F 8D 83`	🍣

Non-integer Numbers

In future lectures we will talk about representing non-integer data. These are called fixed-point and floating-point data types, which we will cover soon!

Preview: How is data actually stored in a program?

C defines a set of standard data types to store information. Each datatype has a specific representation, which depends on the compiler and the CPU architecture.

For the MSP430 architecture, the standard datatypes are defined as follows:

```
int a;              // 16-bit two's-complement signed (2 bytes)
unsigned int b;     // 16-bit unsigned integer (2 bytes)
long int c;         // 32-bit signed integer (two's complement) (4 bytes)
char d;             // 8-bit unsigned integer (1 byte)

float e;            // 32-bit IEEE754 single-precision floating point value (4 bytes)
double f;           // 64-bit IEEE754 double-precision floating point value (8 bytes)
```

DEPENDS ON ARCH.

Note that the types `char`, `float`, and `double` have the same size on all architectures–these are part of the C standard.

We can use these standard datatypes to hold different kinds of information (signed/unsigned numbers, characters, decimal values), or compose more complex types (like arrays or structs).

Important: The size and type of a variable define the range of values they can represent!

- The value of a variable CANNOT exceed the fixed size of the variable
- Variables will "overflow" or "roll over" if the value exceeds the variable size!

INT = 2^{16} POSSIBLE STATES, FOR UNSIGNED

$0 - (2^{16} - 1) = 0 - 6553$

CHAR = 2^{8} " "

$0 - (2^{8} - 1) = 0 - 255$

CHAR C = 253

C = C+1; //254 → 1111 1110

C = C+1; //255 1111 1111

+ 1111 1111

1

1 0000 0000

C = C+1; // 0 0000 0000

=> OVERFLOW